Instanton contribution to the pion and proton electro-magnetic formfactors at $Q^2 \gtrsim 1\,\mathrm{GeV}^2$

P. Faccioli[1*], A. Schwenk[2] and E.V. Shuryak[3†]

[1] *E.C.T.*, Strada delle Tabarelle 286, Villazzano (Trento), I-38050, Italy*

[2] *Department of Physics, The Ohio State University, Columbus, OH 43210, USA*

[3] *Department of Physics and Astronomy, State University of New York, Stony Brook, NY 11794-3800, USA*

Studying the instanton-induced contributions to various hard exclusive reactions provides physical insight into the transition from the non-perturbative to the perturbative regime of QCD. To this end, we compute the leading-instanton contributions to the electro-magnetic and transition formfactors using an effective theory of the instanton liquid model. We report predictions for the electro-magnetic formfactor $F_\pi(Q^2)$ of the pion as well as novel results for the proton Dirac formfactor $F_1(Q^2)$.

I. INTRODUCTION

The transition from the non-perturbative to the perturbative regime of QCD is fundamental to our understanding of the strong interactions. For this purpose, the study of exclusive electro-magnetic reactions at intermediate and high momenta plays a prominent role. As for structure functions from inclusive deep inelastic processes, elastic and transition formfactors encode valuable information about the short-distance structure of the hadrons. In contrast, however, exclusive reactions are sensitive to the non-perturbative forces responsible for the recombination of the scattered partons into the final state.

It is now becoming clear that the asymptotic perturbative regime is not yet reached in the electro-magnetic formfactors, even at surprisingly large momentum transfers. This conclusion follows form the results of two high-precision experiments performed at Jefferson Laboratory (JLAB), namely the measurement of the formfactor of the charged pion and of the ratio of electric to magnetic formfactors $G_E(Q^2)/G_M(Q^2)$ for the proton. The pion formfactor has been measured very accurately for momentum transfers $0.6\,\mathrm{GeV}^2 < Q^2 < 1.6\,\mathrm{GeV}^2$ by the F_π collaboration [1]. Not only are the data at highest experimentally accessible momenta still very far from the asymptotic limit, but also the trend is away from the perturbative QCD (PQCD) prediction (see Fig. 1). This observation contrasts the result of the CLEO experiment for the $\gamma\gamma^\star\pi^0$ neutral pion transition formfactor, where the asymptotic PQCD regime is reached already for $Q^2 \sim 2\,\mathrm{GeV}^2$ [2].

The ratio $\mu\, G_E(Q^2)/G_M(Q^2)$ for the proton (μ is the magnetic moment) has been obtained from recoil polarization measurements up to $Q^2 < 5.6\,\mathrm{GeV}^2$ [4, 5]. At low momenta, this ratio approaches 1, supporting the older SLAC results, which lead to the conclusion that the proton electric and magnetic formfactors can be very well described by the same dipole fit, $G^p_{E(M)\,dip} = e(\mu)/(1+Q^2/M^2_{dip})^2$ with $M_{dip} = 0.84$ GeV. However, at larger momenta, $Q^2 \gtrsim 2\,\mathrm{GeV}^2$, the electric formfactor falls off faster than the magnetic one, and the ratio $\mu\, G_E(Q^2)/G_M(Q^2)$ decreases significantly, in disagreement however with results based on the Rosenbluth separation method.

The slope of the ratio $\mu\, G_E(Q^2)/G_M(Q^2)$ versus Q^2 is sizable and indicates that the asymptotic prediction is not supported by the available experimental data. In fact, the naive Q^2 power counting predicts the same scaling in Q^2 for both electric and magnetic formfactors.

It is natural to ask what dynamics is responsible for the deviation from the asymptotic behavior. By comparing the formfactor of the charged pion to the neutral pion transition formfactor, one concludes that strong non-perturbative dynamics is at work in the former reaction and that it is much weaker in the transition formfactor. Such dynamics must be responsible for delaying the onset of the perturbative regime in the formfactor of the charged pion.

An important question to address is to what extent the observed behavior of the pion and proton formfactors can be understood in terms of the non-perturbative dynamics associated with the spontaneous breaking of chiral symmetry. Clearly, one expects that these forces should play a prominent role in the pion formfactor, due to its Goldstone boson nature. They similarly influence the proton formfactors. This is most evident in the small momentum regime, where a dynamically generated quark mass provides a source for quark helicity flip, therefore contributing to the Pauli formfactor $F_2(Q^2) = (G_M(Q^2) - G_E(Q^2))/(1 + Q^2/(4M^2))$.

---

* Talk given at the 2nd International Conference on Nuclear and Particle Physics at Jefferson Laboratory, Dubrovnik, May 2003.

†Electronic address: faccioli@ect.it, aschwenk@mps.ohio-state.edu, shuryak@tonic.physics.sunysb.edu

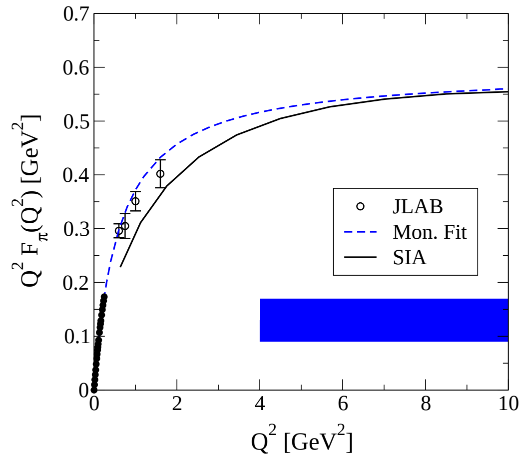

FIG. 1: The recent JLAB data for $Q^2\,F_\pi(Q^2)$ [1] in comparison with the asymptotic PQCD prediction (thick bar, for a typical $\alpha_s \approx 0.2 - 0.4$), the monopole fit (dashed line), and the results of the SIA prediction (solid line). The SIA calculation is not reliable below $Q^2 \sim 1\,\mathrm{GeV}^2$. The solid circles at low Q^2 denote the SLAC data [3].

Instantons are non-perturbative gauge configurations which have been shown to play an important role in the dynamical breaking of chiral symmetry ([6, 7] for a recent study see also [8]), as they naturally generate a density of quasi-zero modes of the Dirac operator. These gluon fields are related to the tunneling between degenerate vacua of QCD. In order to account for their contribution to the correlation functions in QCD, one needs to know how often a tunneling event occurs (i.e., the typical instanton density, $\bar{n}$) and how long it lasts for (i.e., the typical instanton size, $\bar{\rho}$). In the Instanton Liquid Model (ILM), these parameters are extracted phenomenologically from the global properties of the QCD vacuum [9]. This leads to $\bar{n} \simeq 1\,\mathrm{fm}^{-4}$ and $\bar{\rho} \simeq 1/3\,\mathrm{fm}$.

The instanton contribution to the pion and nucleon formfactors has been the subject of several studies over the last few years. Early analyses, however, were either model-dependent or could only make indirect contact with experiments. In [10], Forkel and Nielsen computed the pion formfactor in a sum-rule approach, which takes into account the instanton contribution in the operator product expansion. As in sum-rule approaches, this calculation required a phenomenological description of the continuum of excitations. In order to avoid this model dependence, the relevant electro-magnetic pion and proton three-point functions were calculated in coordinate space in [11, 12]. The result can then be compared to phenomenological estimates of the same correlation functions, obtained by Fourier transforming fits of the experimental data. In this way, the unknown contribution from the continuum of excitations could be excluded by considering sufficiently large-sized correlation functions. Unfortunately, this method has the shortcoming that it does not allow a direct comparison of the theoretical predictions in coordinate space to the experimental data.

A direct comparison between ILM predictions and experiments became possible by combining the Single-Instanton-Approximation (SIA) developed in [13, 14] with the mixed time-momentum representation widely used in lattice calculations [15]. This lead to predictions for the pion and nucleon formfactors [16, 17], and the pion and nucleon correlators, with particular attention to their energy dispersions [18]. The SIA is an effective theory of the ILM, in which the degrees of freedom of the instanton closest to the propagating quarks are treated explicitly, while the contribution of all other pseudo-particles in the vacuum is encoded in a single parameter, the quark effective mass [13]. The combined framework has the advantage that one can compute correlation functions in momentum space, which include the dominant short-distance effects. Moreover, this approach does not have the above discussed model dependences. Clearly, in the SIA it is possible to compute accurately only correlation functions which are dominated by single-instanton effects. This, for example, prevents one from making predictions in the small-momentum regime, typically below 1 GeV (for a detailed discussion see [18]).

II. PION AND NUCLEON FORMFACTORS IN THE SIA

The details of the calculation of the formfactors in the SIA are given in [16, 17]. The formfactors are obtained from appropriate ratios of three- to two-point correlation functions. For example, the pion formfactor is given by

$$\frac{G_4^{(3)}(t, \mathbf{q}/2; -t, -\mathbf{q}/2)}{G^{(2)}(2t, \mathbf{q}/2)} \to F_\pi(Q^2), \tag{1}$$

where $G_4^{(3)}$ denotes the pion three-point correlator with fourth component of the electro-magnetic current. In the time-momentum representation, the three-point function $G_\mu^{(3)}$ is given by

$$G_\mu^{(3)}(t, \mathbf{p}+\mathbf{q}; -t, \mathbf{p}) = \int d^3\mathbf{x}\, d^3\mathbf{y}\, e^{-i\,\mathbf{p}\cdot\mathbf{x}+i\,(\mathbf{p}+\mathbf{q})\cdot\mathbf{y}} \langle 0|\, j_5(t,\mathbf{y})\, J_\mu(0,\mathbf{0})\, j_5^\dagger(-t,\mathbf{x})\,|0\rangle. \tag{2}$$

In the ratio for the formfactor, Eq. (1), the propagation of a pion which is struck by a virtual photon is normalized by the propagation in the absence of the external probe. To this end, one needs the pion two-point correlator $G^{(2)}$, which is defined analogously

$$G^{(2)}(2t, \mathbf{p}) = \int d^3\mathbf{x}\, e^{i\,\mathbf{p}\cdot\mathbf{x}}\, \langle 0|\, j_5(t,\mathbf{x})\, j_5^\dagger(-t,\mathbf{0})\,|0\rangle. \tag{3}$$

Here, the pseudo-scalar current $j_5(x) = \bar{u}(x)\, i\gamma_5\, d(x)$ excites states with the quantum numbers of the pion and $J_\mu(0)$ denotes the electro-magnetic current operator. Similar expressions lead to the nucleon formfactors [17]. We also note that this method does not require the wave function of the hadron as input.

Let us first discuss the results obtained for the pion formfactor [16]. The SIA prediction is shown in Fig. 1 in comparison to the JLAB data and the monopole fit. For momenta where the SIA is a valid approximation, we observe that our results are consistent with the available experimental data, and coincide with the monopole form in the kinematic region accessible to JLAB. This result complements the analysis of Blotz and Shuryak [11], where a similar agreement with the monopole fit was found at small momentum transfers. Clearly, upcoming measurements at JLAB will be able to test the single-instanton prediction as the microscopic non-perturbative mechanism at intermediate momentum transfers.

We find that the single-instanton contribution remains well above the PQCD prediction, throughout the region under present and upcoming investigations at JLAB. This nicely contrasts the situation for the $\gamma\gamma^\star\pi^0$ transition formfactor, where the asymptotic PQCD regime is reached already for $Q^2 \sim 2\,\mathrm{GeV}^2$ [2]. This striking difference is explained by instanton arguments as well, since the three-point function in this case has a different chiral structure. As a result, the instanton contribution to the $\gamma\gamma^\star\pi^0$ reaction is suppressed with respect to the corresponding contribution to the formfactor of the charged pion. Physically, this is the same reason why the vector and axial channels have a rather strong "Zweig" rule, forbidding flavor mixing, while for the pseudo-scalars such a mixing is very strong.

Next, we consider the results for the formfactors of the proton. In [17], the electric formfactor was computed in the same approach. We found that the SIA prediction roughly followed the dipole fit. In order to extend the calculation to the magnetic formfactor, one needs to identify a suitable three-point function which relates directly to the physical formfactor and also receives a maximally enhanced single-instanton contribution. The correlation function satisfying these requirements is given by

$$G_2^{(3)\,M}(t, \mathbf{q}/2; -t, -\mathbf{q}/2) = \int d^3\mathbf{x}\, d^3\mathbf{y}\, e^{i\,\mathbf{q}\cdot(\mathbf{x}+\mathbf{y})/2} \langle 0|\, \mathrm{Tr}[\, \eta_{sc}(t,\mathbf{y})\, J_2(0,\mathbf{0})\, \bar{\eta}_{sc}(-t,\mathbf{x})\, \gamma_2\,]\,|0\rangle, \tag{4}$$

where $\mathbf{q}$ is chosen along the $\widehat{\mathbf{1}}$ direction and $\eta_{sc}(x) = \epsilon^{abc}\,[u_a^T(x) C\gamma_5 d_b(x)]\, u_c(x)$ is the nucleon scalar current, which excites states with the quantum numbers of the proton (C is the charge conjugation matrix).

Although the SIA leads to reasonable results for both formfactors, it turns out that $G_E(Q^2)$ and $G_M(Q^2)$ receive rather large contributions from many-instanton effects, even for relatively high momenta, $Q^2 \sim 1-4\,\mathrm{GeV}^2$, where one would naively expect the single-instanton contribution to be dominant. However, by analyzing the SIA contribution to the relevant three-point correlators, we have found that the single-instanton contribution to the Dirac formfactor $F_1(Q^2) = (G_E(Q^2) + Q^2/(4M^2)\, G_M(Q^2))/(1 + Q^2/(4M^2))$ is enhanced and gives a realistic $F_1(Q^2)$ already for $Q^2 \gtrsim 1\,\mathrm{GeV}^2$, while the SIA contribution to the Pauli formfactor $F_2(Q^2)$ is suppressed. As a consequence, in the SIA one should only expect to obtain a realistic prediction for the electric and magnetic formfactors in the kinematic region where the Dirac formfactor dominates over the Pauli formfactor.

The SIA prediction for $F_1(Q^2)$ is presented in Fig. 2. These are preliminary novel results. The details of this calculation will be reported in a separate publication [19]. The agreement between the theoretical calculation and the extracted experimental data is striking. This implies that the 't Hooft interaction is able to account for the important non-perturbative dynamics in the proton.

Finally, we qualitatively discuss the instanton contributions to the Pauli formfactor $F_2(Q^2)$ of the proton, which receives large contributions from two- and more instantons and thus cannot be addressed in the SIA. The instanton contributions to the Pauli formfactor arise from these collective many-instanton effects, which scale with the square root of the instanton diluteness. The dominant of such instanton-induced collective phenomena is the dynamical breaking of chiral symmetry, which gives rise to a momentum-dependent effective quark mass. Moreover, it has been recently shown by Kochelev that quarks in the instanton vacuum acquire an intrinsic Pauli formfactor [20]. It was

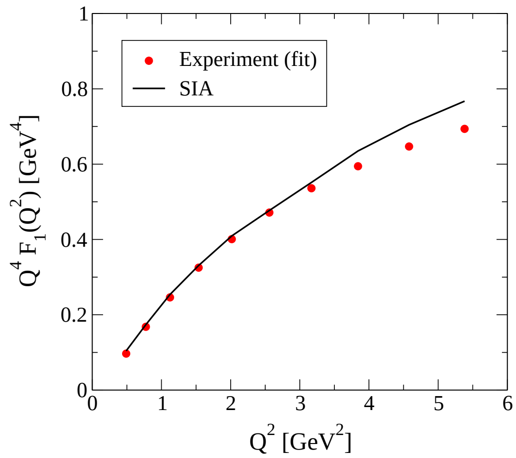

FIG. 2: The SIA prediction for the proton Dirac formfactor (solid line) compared to a fit of the experimental data (circles). The fit has been obtained by assuming a traditional dipole fit for the magnetic formfactor, $G_M(Q^2) = \mu/(1 + Q^2/0.71)^2$, and then extracting $G_E(Q^2)$ from the JLAB parametrization of the electric over magnetic formfactor ratio: $\mu\, G_E(Q^2)/G_M(Q^2) = 1 - 0.13(Q^2 - 0.04)$, with Q in GeV. Asymptotic PQCD gives $Q^4\, F_1(Q^2) \sim$ const [22].

observed that the size of the constituent quarks and the sign of their formfactor agree with the values extracted phenomenologically by Petronzio *et al.* [21]. Both quark effective masses and the Pauli formfactor of the constituent quarks enhance the helicity-flipping transitions, and therefore contribute to $F_2(Q^2)$. A realistic assessment of the contributions of many-instanton effects to the Pauli formfactor of the proton requires challenging numerical simulations in the ILM, which are currently being performed.

III. CONCLUSIONS

We have reported our recent results for the instanton contribution to the pion and proton electro-magnetic formfactors. These are the lightest meson and baryon in the spectrum, where the instanton-induced effects are largest. The SIA makes it possible to perform the calculations for the formfactors directly in momentum space. Our results depend only on two phenomenological parameters of the ILM, which are fixed from global properties of the QCD vacuum.[1] The SIA results agree with experiment, where the corresponding correlators receive contributions from a single instanton. This is the case for the electro-magnetic formfactor of the pion and the Dirac formfactor of the proton. Moreover, the SIA makes predictions for the kinematic regime under investigation at JLAB and constrains the onset of the asymptotic PQCD regime to relatively high momenta for the pion formfactor. Our results imply that instanton-induced forces are able to account for the electro-magnetic structure of the light hadrons at short distances.

Studying the instanton-induced contribution to the hadron formfactors is also important from a broader standpoint, in order to understand the transition from the non-perturbative to the perturbative regime of QCD. We have shown that the 't Hooft interaction provides an explanation why the perturbative regime is reached much later in the pion formfactor than for the $\gamma\gamma^*\pi_0$ transition formfactor. In order to study the effect of instantons on the electro-magnetic formfactor of the proton in the same transition window as for the pion formfactor, one has to evaluate the Pauli formfactor, $F_2(Q^2)$, in the ILM. This problem is currently under numerical investigation.

[1] J. Volmer *et al.*, Phys. Rev. Lett. **86** (2001) 1713.
[2] J. Gronberg *et al.*, Phys. Rev. **D57** (1998) 33.
[3] S.R. Amendolia *et al.*, Nucl. Phys. **B277** (1986) 168.
[4] M.K. Jones, *et al.*, Phys. Rev. Lett. **84** (2000) 1398.

[1] Note that the SIA prediction of the pion formfactor does not depend on the instanton density, but only on the instanton size, which is extracted from lattice simulations

[5] O. Gayou, *et al.*, Phys. Rev. Lett. **88** (2002) 092301.
[6] D. Diakonov, Chiral Symmetry Breaking by Instantons, Lectures given at the Enrico Fermi School in Physics, Varenna, 1995, hep-ph/9602375.
[7] T. Schäfer and E.V. Shuryak, Rev. Mod. Phys. **70** (1998) 323.
[8] P. Faccioli and T.A. DeGrand, Phys. Rev. Lett. (2003) in press, hep-ph/0304219.
[9] E.V. Shuryak, Nucl. Phys. **B214** (1982) 237.
[10] H. Forkel and M. Nielsen, Phys. Lett. **B345** (1997) 55.
[11] A. Blotz and E.V. Shuryak, Phys. Rev. **D55** (1997) 4055.
[12] P. Faccioli and E.V. Shuryak, Phys. Rev. **D65** (2002) 076002.
[13] P. Faccioli and E.V. Shuryak, Phys. Rev. **D64** (2001) 114020.
[14] P. Faccioli, Ph.D. Thesis, SUNY Stony Brook, 2002, unpublished.
[15] T. Draper, R.M. Woloshyn, W. Wilcox and K.F. Liu, Nucl. Phys. **B318** (1989) 319.
[16] P. Faccioli, A. Schwenk and E.V. Shuryak, Phys. Rev. **D67** (2003) 113009.
[17] P. Faccioli, A. Schwenk and E.V. Shuryak, Phys. Lett. **B549** (2002) 93.
[18] P. Faccioli, Phys. Rev. **D65** (2002) 094014.
[19] P. Faccioli and E.V. Shuryak, in preparation.
[20] N.I. Kochelev, Phys. Lett. **B565** (2003) 131.
[21] R. Petronzio, S. Simula and G. Ricco, Phys. Rev. **D67** (2003) 094009.
[22] S.J. Brodsky and G.R. Ferrar, Phys. Rev. Lett. **31** (1973) 1153.
G.P. Lepage and S.J. Brodsky, Phys. Rev. Lett. **43** (1979) 545.